Handling qualities for Bicycles and Motorcycles, a preview.

Extracted from "The Chronicles of the Lords of the Chainring"

For a better understanding of these equations, the simplifications and their applications, see Lords of the Chainring. The text is available from Prof. Patterson at wpatters@calpoly.edu

This is the study of the response of a 2 wheeled vehicle to displacement of and or force on the controls. It is a radical departure from, hands free stability, which is the classical method of analyzing bicycle dynamics.

Early literature has not emphasized the importance of trail to a vehicle's feedback. In fact, early stability plots were presented in terms on non-dimensional geometry. This may be normal procedure for scientists because non-dimensional numbers translate to the general condition. The problem with this approach is that a myth of castor angle was born. Bike designers assumed that only front wheel size should determine trail.

A very complicated computer model was developed to predict bike response. Vehicle state plays a significant role in it's dynamics. The bike was placed in a constant -speed – constant- rate turn, and it's response to control inputs was determined. It was then discovered that the response of a bike in a straight line is very similar to the response of a bike in a curving path. The computer model was used to isolate the second and third order dynamic terms that contributed little to the bike's response. The goal was to reach that "Golden Fleece" of the engineer, a simplified design tool.

Placing the bike in a straight line and making suitable simplifications, allowed the derivation of 3 equations. These equations provide the designer a method to formulate suitable head tube angle and trail for widely varied bike configurations.

The tool describes:

1. The control spring
2. The minimum trail necessary to control high speed over control.
3. The maximum trail to restrain fork flop below oppressive levels.

This tool, and it's predecessors, have been used to design and build more than 200 student bikes at the University and they have also been used to design 3 production bikes.

Symbols

A	wheelbase
B	horizontal position of the center of gravity
h	height of the center of gravity

Kx radius of gyration through the cg.
M mass of bike and rider
β complement of the head tube angle
δ Steering angle.
Nf Normal force at the front wheel Wt B/A
ΔQ Change in control torque
R Front wheel radius
Rh Radius of the handlebars
S Fork offset
T Trail $(R \sin(\beta)-S)/\cos(\beta)$
Wt Weight of the bike and rider.
• Multiplication

Control spring.

The control spring is felt as a torque transmitted up the steer tube. The normal equation for a torsional spring is:

$$\Delta Q = - K \Delta\delta$$

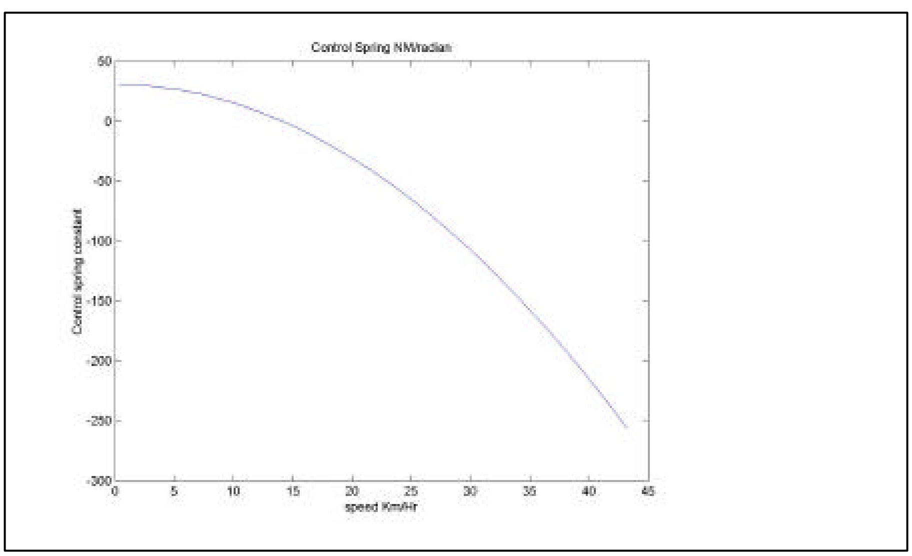
The change in spring torque is supposed to oppose any angular deflection. However, the spring constant for a bicycle changes with velocity. So that

$$\Delta Q = (K1-(K2 *V^{2})) \Delta\delta$$

The bicycle effective spring constant is $(K1-K2 *V^{2})$.

A bicycle suffers from the strange condition of having a positive spring at low speed. Instead of opposing your hands at low speed, the handlebars tend to continue in the direction of the turn.

CONTROL SPRING CONSTANT VS SPEED



$K1 = Nf * \mathbf{T} * \cos(\beta) * \sin(\beta)$

The designer may need to solve problems with low speed control. If this is the case, Sin(β) can be minimized by making the head tube as vertical as practical, thereby reducing K1.

Minimum Trail

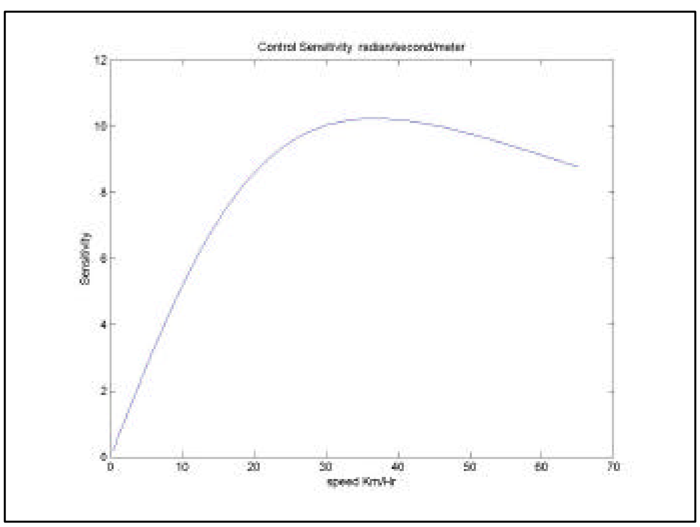
Trail provides the feedback torque that mitigates the "over-control" condition found in too many bikes today. The position of the center of gravity and the longitudinal radius of gyration through the center of gravity determine the minimum trail necessary to control sensitivity.

$$Tmin = K5 * (B/M)(1/h^2 + 1/Kx^2)$$

Note: K5 is applicable for mass measured in Kilograms. B, h, and Kx are measured in Meters.

K5 has the value of 1.2 kg m^2 for light steering up to 2.4 for heavier, more normal steering.

CONTROL SENSITIVITY VS SPEED



Maximum trail

A bike with too much trail, will have problems with fork flop. The fork will forcibly turn in the direction of frame tilt. Flop ranges from 75 to 200 n/rad for nice handling bikes. A maximum of something less that 300 seems to be fine. Then let

Flop = 275

$$Tmax = Flop * Rh/(Nf * \cos(\beta))$$

DESIGN PROCESS

The designer can use these equations to make 3 adjustments to his machine. The first is to move the seat backward or forward to change "B". The second to set appropriate head tube angle. The third is to bend to fork backward or forward to change trail by changing fork offset "S". The designer can have almost any configuration of bike and change B, β and S to give proper handling.

The important equations for high-speed equanimity are:

$$Trail = (R \sin(\beta)-S)/\cos(\beta)$$

$$\text{Minimum Trail} = K5* (B/M)(1/h^2 + 1/Kx^2)$$